Solutions for Logictionary Game

Here's one set of solutions for the Logictionary Game we played in class. Did you end up with something else? Feel free to post on Piazza and we can take a look!

i. Given the predicates *Orange*(x), which states that x is orange, and *Cat*(x), which states that x is a cat, write a formula in first-order logic that says "every cat is orange."

> This nicely matches one of our Aristotelian forms: $\forall c.\ (Cat(c) \to Orange(c))$.

ii. Given the predicates *Orange*(x), which states that x is orange, and *Cat*(x), which states that x is a cat, write a formula in first-order logic that says "some cat is orange."

> Another Aristotelian form: $\exists c.\ (Cat(c) \land Orange(c))$.

iii. Given the predicates *Orange*(x), which states that x is orange, and *Cat*(x), which states that x is a cat, write a formula in first-order logic that says "there are no orange cats."

> Yet another Aristotelian form! $\forall c.\ (Cat(c) \to \neg Orange(c))$.

iv. Given the predicates *Orange*(x), which states that x is orange, and *Cat*(x), which states that x is a cat, write a formula in first-order logic that says "some cat is not orange."

> The last of Aristotelian forms: $\exists c.\ (Cat(c) \land \neg Orange(c))$.

v. Given the predicates *Person*(x), which states that x is a person; *Orange*(x), which states that x is orange; *Cat*(x), which states that x is a cat; and *Likes*(x, y), which states that x likes y, write a formula in first-order logic that says "everyone likes at least one orange cat."

> If we go step by step through a translation and use the Aristotelian forms as a guide, we get this:
>
> $$\begin{aligned}&\forall p.\ (Person(p) \to \\ &\quad \exists c.\ (Cat(c) \land Orange(c) \land Likes(p, c)) \\ &)\end{aligned}$$
>
> This says "for any person p, there's a cat c that they like."

vi. Given the predicates *Person*(*x*), which states that x is a person; *Cat*(*x*), which states that *x* is a cat; and *Likes*(*x*, *y*), which states that x likes y, write a formula in first-order logic that says "everyone likes ***exactly one*** cat."

Using a combination of the Aristotelian forms and what we saw about uniqueness in lecture, we can come up with something like this:

$$\begin{aligned}&\forall p.\ (Person(p) \to \\ &\quad \exists c.\ (Cat(c) \land Likes(p, c) \land \\ &\quad\quad \forall d.\ (Cat(d) \land d \neq c \to \neg Likes(p, d)) \\ &\quad) \\ &)\end{aligned}$$

This says "for any person *p*, there's a cat *c* that they like, and that person *p* doesn't like any other cats."

vii. Given the predicate *Person*(*x*), which states that *x* is a person, and *Muggle*(*x*), which states that *x* is a muggle, write a statement in first-order logic that says "some (but not all) people are muggles."

One possibility is given here, which says both that someone is a muggle and that someone is not a muggle.

$$\exists p.\ (Person(p) \land Muggle(p)) \quad \land \quad \exists p.\ (Person(p) \land \neg Muggle(p))$$

Another equivalent option is to say that someone is a muggle and that it's not the case that everyone is a muggle. Here's one way to do that:

$$\exists p.\ (Person(p) \land Muggle(p)) \quad \land \quad \neg\forall p.\ (Person(p) \to Muggle(p))$$

viii. Given the predicate *Person*(*x*), which states that *x* is a person, and *Ruler*(*x*), which states that *x* is a ruler, write a statement in first-order logic that says "there is at most one ruler."

One way to express this idea is to say that either no one is a ruler, or there is just one person who's a ruler and everyone else isn't a ruler. This is shown here:

$$\begin{aligned}&\neg\exists p.\ (Person(p) \land Ruler(p)) \lor \\ &\exists p.\ (Person(p) \land Ruler(p) \land \\ &\quad \forall q.\ (Person(q) \land p \neq q \to \neg Ruler(q)) \\ &)\end{aligned}$$

Another option which works, but is a lot more subtle, is to say that there's some person where everyone who is a ruler is that one person. That way, if the person isn't a ruler, then no one is a ruler, and if that person is a ruler, then no one else is. This is shown here:

$$\begin{aligned}&\exists p.\ (Person(p) \land \\ &\quad \forall q.\ (Person(q) \land Ruler(q) \to p = q) \\ &)\end{aligned}$$

This second one is a lot trickier to come up with and it's pretty subtle to see why it's correct, so don't worry if you didn't think of it. We thought we'd include it just for completeness' sake.